Nuclear Binding & Nuclear Structure

Because energy must be added to a nucleus to separate it into its individual constituents, (n's, p's), the total rest mass (and energy) E_0 of the separated nucleons is greater than the rest mass (energy) of the nucleus. Thus the rest energy of the nucleus is $E_0 - E_B$

$\uparrow$ binding energy ($\frac{E_B}{c^2}$: Mass defect)

For a nucleus with Z: protons, N: neutrons

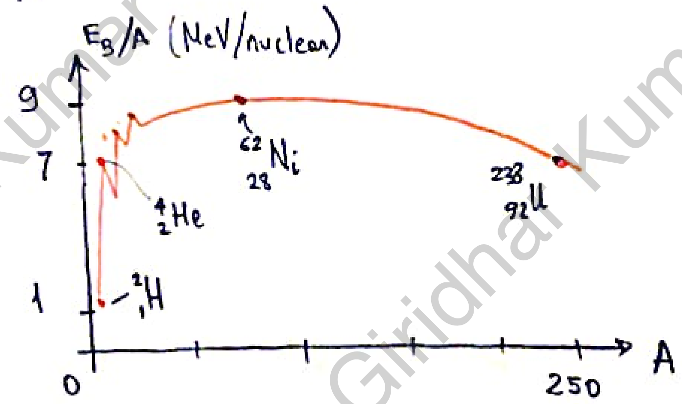
$$E_B = \left(Z M_H + N m_n - \underbrace{{}^{A}_{Z}M}_{\text{mass of neutral atom}}\right) c^2$$

Ex: The nucleus of deuterium ${}^2_1H$ is called deuteron: (p)(n)

$$E_B = \left(1.007825\,u + 1.008665\,u - 2.014102\,u\right) \frac{931.5\ \text{MeV}}{u}$$

$$= 2.224\ \text{MeV}$$

$$\frac{E_B}{A} = 1.112\ \text{MeV}$$ ... binding energy per nucleon (a more universal parameter)



${}^2_1H$ has the lowest binding en. per nucleon of all nuclides, whereas nickel ${}^{62}_{28}Ni$ has the highest. α-particle also has quite high value!

Nuclear Force (No simple Coulomb's Law-like expression is currently available!)

The force that binds p's and n's together in the nucleus, despite the electrical repulsion of the p's is an example of the strong force.

Ex: Calculate the Coulomb energy bet. two p's separated by 1 fm.

$$E_C = e \cdot \frac{e}{4\pi\epsilon_0 r} = \frac{(1.6\ 10^{-19})^2}{\underbrace{4\pi \cdot 8.8}_{\simeq 100}\ 10^{-12}} \cdot 10^{15} \Big/ \underbrace{(1.6 \cdot 10^{-19})}_{\text{to convert into eV}} \simeq 1.6\ \text{MeV}$$

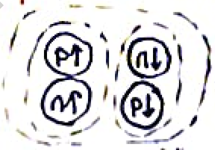
Some Properties of the Nuclear Force:

1) It does not depend on the electric charge (same for n's & p's)

2) It has short range ~ a few fm. Otherwise the nucleus would grow by pulling in additional p's & n's. Within its range, nuclear force is much stronger than electrical forces; otherwise nucleus could be unstable.

3) The nearly constant density of nuclear matter and the nearly const. E_B/A of larger nuclides show that a particular nucleon cannot interact simultaneously with all the other nucleons in a nucleus, but only with those few in its immediate vicinity. (This is in contrast with Coulomb force.) This limited number of interactions is called saturation (analogous to covalent bonding in molecules/solids).

4) Nuclear force favors binding of pairs of p's & n's with opposite spins and of pairs of pairs



Hence, α-particle ${}^{4}_{2}He$ is an exceptionally stable nucleus for its mass number. (This pairing reminds the Cooper pairs in BCS theory)

Nuclear Models

In the nucleus three different kinds of force take part (EM, weak and strong forces). The combination of all three make the nuclear force quite complicated, yet to be fully understood. For this reason the analysis of nuclear structure is more complex than the analysis of many-electron atoms.

Several simple nuclear models exist which are of great help, each with different levels of success, in gaining some insight into nuclear structure.

Liquid-Drop Model

* First proposed by George Gamow in 1928, later expanded by N. Bohr.
* Based on the observation that all nuclei have nearly same density
* Quite successful in correlating nuclear masses, and understanding decay processes of unstable nuclides
* Other models are better suited for angular momentum and excited state properties.
* This is a phenomenological model which does not use a QM framework (unlike the Shell Model)

In the Liquid-Drop model, a nucleus with mass number A and atom number Z has the following form of binding energy.

$$E_B = \underbrace{C_1 A}_{\text{volume (saturation)}} - C_2 A^{2/3} - C_3 \frac{Z(Z-1)}{A^{1/3}} - C_4 \frac{(A-2Z)^2}{A} \pm C_5 A^{-4/3}$$

volume (saturation): An individual nucleon only interacts with its nearest neighbors

$\Rightarrow E_B \propto A$ ← # nucleons

Surface: Nucleons on the surface are less tightly bound (no neighbors outside)

$-4\pi R^2$ term

$R \propto A^{1/3}$

$\Rightarrow -A^{2/3}$

Coulomb: every proton (Z of them) repels the others (Z-1)

$$-\frac{Z.(Z-1)}{R}$$

$R \to A^{1/3}$

Balance:

$E_B \propto \frac{(N-Z)^2}{A}$ ... gives a good balance bet. the energies associated w/ n's & p's

pairing:

Nuclear force favors pairing of p's & n's

Choose + sign: if both Z & N are even

Choose - sign: " " " " " odd

Set to zero: otherwise

Best fit is obtained with constants taken as:

$C_1 = 15.75$ MeV, $C_2 = 17.80$ MeV, $C_3 = 0.7100$ MeV, $C_4 = 23.69$ MeV, $C_5 = 39$ MeV

Combining this expression with $E_B = (ZM_H + Nm_n - {}^A_Z M)c^2$ leads to

$${}^A_Z M = ZM_H + Nm_n - \frac{E_B}{c^2}$$ ... semiempirical mass formula

Example: E_B and M estimation for $^{62}_{28}Ni$ based on Liquid-Drop Model

$Z = 28, \quad A = 62, \quad N = 34$

The five terms in the LDM have the following contributions:

1. $C_1 A = 976.5$ MeV

dominant contributions

2. $-C_2 A^{2/3} = -278.8$ MeV

3. $-C_3 \frac{Z(Z-1)}{A^{1/3}} = -135.6$ MeV

4. $-C_4 \frac{(A-2Z)^2}{A} = -13.8$ MeV

5. $+ C_5 A^{-4/3} = 0.2$ MeV

Both N & Z are even

$\rightarrow E_B = 548.5$ MeV $\xrightarrow[\text{LDM with}]{\text{compare}}$ 545.3 MeV (true value)

LDM is only 0.6% larger than the true value

Use E_B in semiempirical mass formula

$$^{62}_{28}M = 28(1.007825u) + 34(1.008665u) - \frac{548.5}{931.5} = 61.925u$$

only 0.005% smaller than

61.928349 u (true value)